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AN EXACT TRANSFORMATION FROM GEOCENTRIC TO GEODETIC COORDINATES FOR NONZERO ALTITUDES

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16. Abstract

An exact method for the nonzero altitude transformation from geocentric to geodetic coordinates is derived. The method is mathematically general and should serve as a primary standard.

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INTRODUCTION

In space position measurement systems, it is often necessary to transform geocentric to geodetic coordinates. Consequently, it is important to have an exact closed-form solution for the transformation that is free of embedded singularities. An exact solution is particularly important for determining missile positions. All the previous work in this area falls into two categories: formulas that are mathematically exact but involve equations that become unstable in the neighborhood of their singularities, and formulas that are approximations that may or may not have inherent singularities. References 1 and 2 are of the former type, and references 3 to 5 are of the latter.

Although some of these methods are adequate under some conditions, they have limitations under certain circumstances. This paper presents an exact closed-form solution for the transformation that is free of singularities. The solution permits the evaluation of any of the existing methods under any conditions and should serve as a primary standard.

The appendix contains computer subroutines that implement the ideas that follow and examples of input and output data.

SYMBOLS

Parenthetical symbols are computer identifiers for variables.

a (A)	major axis of ellipsoid
b (B)	minor axis of ellipsoid
d	the distance from any point in space to the ellipsoid
d_x	$= x_0 - x_2$

d_y	$= y_0 - y_2$	
d_z	$= z_0 - z_2$	
e (E)	eccentricity	
f (F)	flattening factor	
$f(x,y,z)$	general function to be minimized	
g	restraint imposed on f	
H	function which is composite of functions f and g	
h (ALT)	altitude normal to the ellipsoid	
h_s (ALTS)	altitude computed from distance formula	
p_0	any point in space	
p_2	the point on the ellipsoid that is the minimum distance from the point p_0	
t_x	percentage of error in d_x	
t_y	percentage of error in d_y	
t_z	percentage of error in d_z	
X	coordinate axis that intercepts the Greenwich meridian	
x,y,z	coordinates of any point on the ellipsoid	
x_0, y_0, z_0 (X0,Y0,Z0)	coordinates of p_0	
x_2, y_2, z_2 (X2,Y2,Z2)	coordinates of p_2	
Y	coordinate axis that is in a direction normal to the plane determined by the intersection of X and Z	
Z	coordinate axis that intercepts the poles	

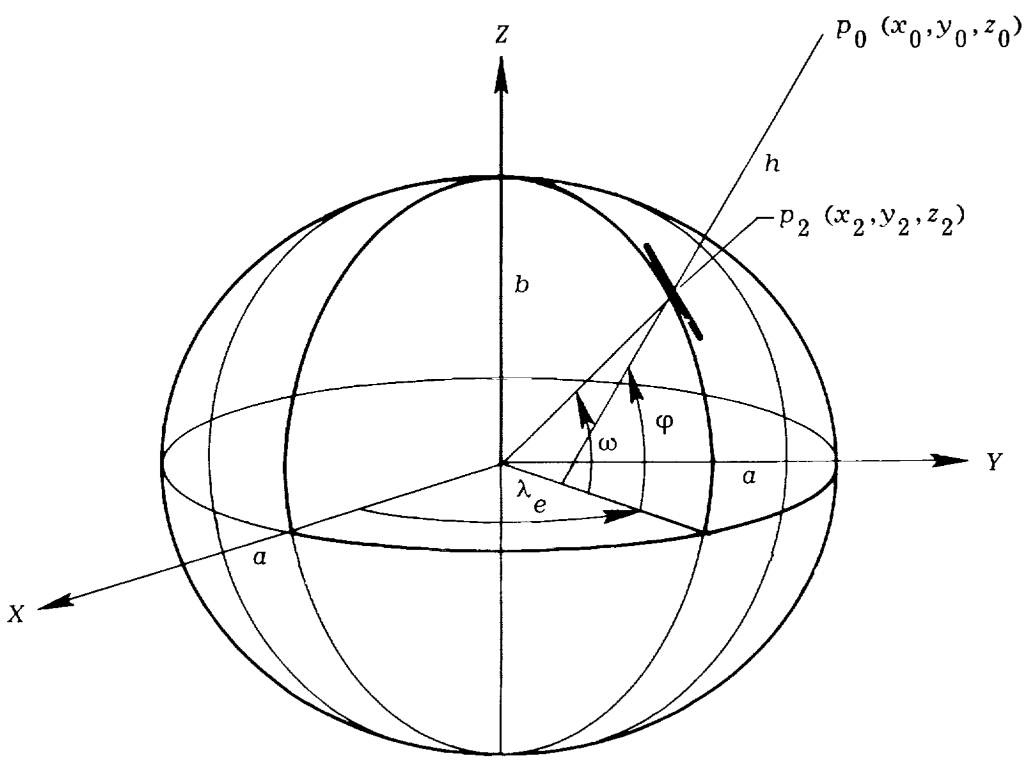
α (V)	arbitrary real constant to be determined
ε	error in α
λ_e (ALONG)	longitude east
λ_w (ALONG)	longitude west
φ (ALAT)	geodetic latitude
ω	geocentric latitude

ANALYTICAL FORMULATION

Let a model of the earth be an ellipsoid given by the following equation (ref. 6):

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \tag{1}$$

where $a > b$. In addition, let p_0 (x_0, y_0, z_0) be any point above the ellipsoid, as shown in the sketch below.



The line perpendicular to the surface of the ellipsoid from p_0 is the altitude of p_0 above the ellipsoid and hence is the shortest distance from p_0 to that surface (eq. (1)).

Therefore, by minimizing the distance from p_0 to the surface of the ellipsoid, it is possible to acquire the coordinates (x_2, y_2, and z_2) of the point on the surface that cause the distance defined by

$$d = \left[\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2\right]^{1/2} \tag{2}$$

to be minimum.

The Lagrange multiplier method (ref. 7) is used to implement this minimization where

$$f(x,y,z) = \left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2$$

and

$$g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1$$

Then

$$H(x,y,z,\alpha) = \left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2 - \alpha\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1\right)$$

If partial derivatives are taken in turn and each is equated to zero, the following equations result:

$$\frac{\partial H}{\partial x} = 2\left(x - x_0\right) - \frac{\alpha 2x}{a^2} = 0 \text{ or } x = x_0/\left(1 - \alpha/a^2\right) \tag{3}$$

$$\frac{\partial H}{\partial y} = 2\left(y - y_0\right) - \frac{\alpha 2y}{a^2} = 0 \text{ or } y = y_0/\left(1 - \alpha/a^2\right) \tag{4}$$

$$\frac{\partial H}{\partial z} = 2\left(z - z_0\right) - \frac{\alpha 2z}{b^2} = 0 \text{ or } z = z_0/\left(1 - \alpha/b^2\right) \tag{5}$$

$$\frac{\partial H}{\partial \alpha} = -\frac{x^2}{a^2} - \frac{y^2}{a^2} - \frac{z^2}{b^2} + 1 = 0 \tag{6}$$

Equations (3) to (5) can be substituted into equation (6) as follows:

$$-\left[x_0/\left(1 - \alpha/a^2\right)\right]^2/a^2 - \left[y_0/\left(1 - \alpha/a^2\right)\right]^2/a^2 - \left[z_0/\left(1 - \alpha/b^2\right)\right]^2/b^2 + 1 = 0$$

or

$$-x_0^2\left(b^2 - 2\alpha + \alpha^2/b^2\right) - y_0^2\left(b^2 - 2\alpha + \alpha^2/b^2\right) - z_0^2\left(a^2 - 2\alpha + \alpha^2/a^2\right) + \left(a^2 - 2\alpha - \alpha^2/a^2\right)\left(b^2 - 2\alpha + \alpha^2/b^2\right) = 0$$

Simplifying and collecting,

$$\begin{aligned} &\left(1/a^2b^2\right)\alpha^4 - 2\left(1/a^2 + 1/b^2\right)\alpha^3 + \left(4 + a^2/b^2 + x_0^2/b^2 - y_0^2/b^2 - z_0^2/a^2\right)\alpha^2 \\ &\quad + 2\left(x_0^2 + y_0^2 + z_0^2 - a^2 - b^2\right)\alpha + a^2b^2 - x_0^2b^2 - y_0^2b^2 - z_0^2a^2 = 0 \end{aligned} \tag{7}$$

This is a quartic equation in α and can be solved in closed form (ref. 8). The proper solution can be determined if the solution is restricted to the real zeros of equation (7) and that zero of equation (7) is chosen that causes d to be smallest. The appropriate solution of this quartic has no singularities. That is, α does not approach plus or minus infinity, because that would imply from equations (3) to (5) that x_2, y_2, and z_2 equal zero. Equations (3) to (5) do not become unstable, because the statement $\alpha = a^2$ or $\alpha = b^2$ implies that x_2, y_2, or z_2 approaches infinity. Since altitude is nonzero, α does not equal zero.

Let α_0 be that zero that satisfies the quartic equation. Then, from equations (3) to (5),

$$x_2 = x_0/\left(1 - \alpha_0/a^2\right) = x_0a^2/\left(a^2 - \alpha_0\right) \tag{8}$$

$$y_2 = y_0/\left(1 - \alpha_0/a^2\right) = y_0a^2/\left(a^2 - \alpha_0\right) \tag{9}$$

$$z_2 = z_0/\left(1 - \alpha_0/b^2\right) = z_0b^2/\left(b^2 - \alpha_0\right) \tag{10}$$

$$h_s = \left[\left(x_2 - x_0\right)^2 + \left(y_2 - y_0\right)^2 + \left(z_2 - z_0\right)^2\right]^{1/2} \tag{11}$$

$$\varphi = \sin^{-1}\left(z_0 - z_2\right)/h_s \tag{12}$$

$$\lambda_e = \tan^{-1}\left(y_2/x_2\right) = \tan^{-1}\left(y_0/x_0\right) = -\lambda_w \tag{13}$$

Although the foregoing formulas are mathematically exact, for computational reasons it is usually advantageous to normalize all distances, because this minimizes errors due to rounding off. However, errors due to rounding off are still possible. To circumvent this problem for altitudes not near zero, consider the following error analysis with respect to the computation of h_s, λ, and φ.

Let ε be a small error introduced in the computation of the zero of the quartic. In addition, let $d_z = z_0 - z_2 = -z_0\alpha_0/\left(b^2 - \alpha_0\right)$. Then the error in d_z is introduced by α_0 only, because a, b, x_0, y_0, and z_0 are all exact. It is easy to show that the percentage of error, t_z, of d_z is as follows:

$$t_z = -1 + \left(1 + \varepsilon/\alpha_0\right)\left[\left(b^2 - \alpha_0\right)/\left(b^2 - \alpha_0 - \varepsilon\right)\right]$$

From equation (10), $b^2 - \alpha_0 > b^2$. Therefore $\left|b^2 - \alpha_0\right| > \left|\alpha_0\right| > \left|\varepsilon\right|$, and since ε is small, $\left(b^2 - \alpha_0\right)/\left(b^2 - \alpha_0 - \varepsilon\right) \approx 1$. Therefore $t_z \approx \varepsilon/\alpha_0$. Similarly, $t_x, t_y \approx \varepsilon/\alpha_0$.

Since equation (12) involves the ratio of two numbers with virtually equal percentages of error, latitude is undisturbed by a small error in the computation of α_0 due to rounding off.

As noted above, this analysis is valid only if the altitude does not approach zero; otherwise α_0 approaches zero and the value $\left|\varepsilon/\alpha_0\right|$ may approach infinity. The instability of $\left|\varepsilon/\alpha_0\right|$ suggests that equation (12) may become unstable under these conditions.

This final problem can be resolved. If $p_2\ (x_2, y_2, z_2)$ is the point on the ellipsoid that intersects the normal drawn from $p_0\ (x_0, y_0, z_0)$, the following equation (ref. 6) may be used:

$$\varphi = \tan^{-1}\left[\tan\omega/\left(1 - e^2\right)\right] \tag{14}$$

where $\tan\omega = z_2/\left(x_2^{\ 2} + y_2^{\ 2}\right)^{1/2}$. From these relationships, φ may be determined immediately. The error, if any, in $\tan\omega$ introduced by the computational round-off error in α_0 is inconsequential, because it can easily be shown that the percentage of error in x_2, y_2, and z_2 is virtually zero. In fact, the percentage of error in z_2 and x_2 or y_2 is given by $\left(b^2 - \alpha_0\right)/\left(b^2 - \alpha_0 - \varepsilon\right) - 1$ and $\left(a^2 - \alpha_0\right)/\left(a^2 - \alpha_0 - \varepsilon\right) - 1$, respectively. Since ε is small for any value of α_0, the quantities

$(b^2 - \alpha_0)/(b^2 - \alpha_0 - \varepsilon)$ and $(a^2 - \alpha_0)/(a^2 - \alpha_0 - \varepsilon)$ approximate 1 under any conditions. Thus, equation (14) is preferable to equation (12) for determining geodetic latitude for any altitude.

However, even the smallest error in α_0 causes more than negligible error in the computation of h_s (eq. (11)), because the error is amplified by the scaling factor used to restore the true value of h after computations are completed. This analysis suggests that it is preferable to use the following exact formulas from reference 6 to compute h instead of equation (11):

$$x_0 = \left[a/\left(1 - e^2 \sin^2 \varphi\right)^{1/2} + h\right] \cos \varphi \cos \lambda$$

$$y_0 = \left[a/\left(1 - e^2 \sin^2 \varphi\right)^{1/2} + h\right] \cos \varphi \sin \lambda$$

$$z_0 = \left[a\left(1 - e^2\right)/\left(1 - e^2 \sin^2 \varphi\right)^{1/2} + h\right] \sin \varphi$$

where $f = (a - b)/a$ and $e^2 = 2f - f^2$. Since latitude and longitude are known, h can be computed directly and the proper equation can be chosen to compute h to avoid division by values close to zero.

In this way the geodetic coordinates φ, λ_w, and h are determined exactly both mathematically and computationally. Therefore, the computational accuracy of the method is the accuracy of the computer used.

CONCLUDING REMARKS

A mathematically exact method for computing geodetic coordinates from geocentric coordinates is derived. The computational accuracy achieved by using the method is as accurate as the computer used. The transformation provides a primary standard and makes possible the evaluation of any of the existing methods.

Flight Research Center
National Aeronautics and Space Administration
Edwards, Calif., November 4, 1975

APPENDIX

COMPUTER SUBROUTINES AND DATA SAMPLES

The following computer subroutines implement the theory presented in the text. The subroutines were written in FORTRAN IV.

The comments in the listings should be sufficient for their comprehension and modification. A sample input and output listing is provided to facilitate the verification of the correct FORTRAN code.

Subroutine GEOD

```
      SUBROUTINE GEOD(ELE,ELEM,ELES,AZI,AZIM,AZIS,PHI,PHIM,PHIS,
     *ALM,ALMM,ALMS,RAN,H,ALAT,ALONG,ALT)
C
C
C     THIS SUBROUTINE COMPUTES THE ALTITUDE, GEODETIC LATITUDE AND
C     LONGITUDE GIVEN THE RANGE, AZIMUTH AND ELEVATION OF A POINT
C     WITH RESPECT TO A RADAR SITE.
C
C
C     ELE,ELEM,ELES ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF
C     ELEVATION OF THE TARGET WITH RESPECT TO A RADAR SITE.
C
C     AZI,AZIM,AZIS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY OF
C     AZIMUTH OF THE TARGET WITH RESPECT TO A RADAR SITE.
C
C     PHI,PHIM,PHIS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY
C     OF THE GEODETIC LATITUDE OF THE RADAR SITE.
C
C     ALM,ALMM,ALMS ARE THE DEGREES, MINUTES AND SECONDS RESPECTIVELY
C     OF THE LONGITUDE OF THE RADAR SITE.
C
C     RAN IS THE RANGE OF THE TARGET WITH RESPECT TO THE RADAR SITE.
C     H IS THE ALTITUDE OF THE RADAR SITE ABOVE THE REFERENCE ELLIPSOID.
C
C     ALAT IS THE COMPUTED GEODETIC LATITUDE.
C     ALONG IS THE COMPUTED LONGITUDE.
C     ALT IS THE COMPUTED ALTITUDE.
C
      DIMENSION R(12),ROOT(12)
C
C     DEFINE MAJOR AND MINOR AXIS OF ELLIPSOID IN FEET.
C
      A=20925832.
      B=20854892.
      F=(A-B)/A
      CE=2*F-F**2
C
C     NORMALIZE IN UNITS OF A.
C
      ZP=A
      A1=A
      H=H/ZP
      RAN=RAN/ZP
      A=A/ZP
      B=B/ZP
      GK=3.1415926536
      CON=GK/180.
C
C        CONVERT ANGLES FROM DEGREES TO RADIANS.
C
      ELE=ELE+(ELEM/60)+(ELES/3600)
      PHI=PHI+(PHIM/60)+(PHIS/3600)
      AZI=AZI+(AZIM/60)+(AZIS/3600)
      ALM=ALM+(ALMM/60)+(ALMS/3600)
      ALM=360-ALM
      PHI=PHI*CON
      ALM=ALM*CON
```

```
      AZI=AZI*CON
      ELE=ELE*CON
C
C
C        COMPUTE THE TRANSFORMATION OF TARGET COORDINATES,
C        FROM POLAR COORDINATES TO CARTESIAN COORDINATES,
C        CENTERED AT TRACKING RADAR.
C
      X0=RAN*SIN(AZI)*COS(ELE)
      Z0=RAN*SIN(ELE)
      Y0=RAN*COS(AZI)*COS(ELE)
C
C        COMPUTE THE TRANSFORMATION OF THE RADAR SITE GIVEN
C        IN GEODETIC POLAR COORDINATES TO CARTESIAN COORDINATES
C        WITH RESPECT TO THE GEOCENTRIC CENTER OF THE EARTH.
C
      EX=1/(1-CE*(SIN(PHI)**2))
      EX=ABS(EX)
      EX=EX**.5
      EX=EX*A
      E0=(EX+H)*COS(PHI)*COS(ALM)
      F0=(EX+H)*COS(PHI)*SIN(ALM)
      G0=(EX*(1-CE)+H)*SIN(PHI)
C
C     COMPUTES THE DIRECTION COSINES OF THE AXIS OF THE RADAR SITE.
C
      U1=-SIN(ALM)
      U2=COS(ALM)
      U3=0
      V1=-COS(ALM)*SIN(PHI)
      V2=-SIN(ALM)*SIN(PHI)
      V3=COS(PHI)
      W1=COS(ALM)*COS(PHI)
      W2=SIN(ALM)*COS(PHI)
      W3=SIN(PHI)
C
C        COMPUTE THE TRANSFORMATION OF THE POINT OR TARGET
C        FROM CARTESIAN COORDINATES CENTERED AT TRACKING
C        RADAR TO GEOCENTRIC CARTESIAN COORDINATES.
C
      X5=X0*U1+Y0*V1+Z0*W1+E0
      Y5=X0*U2+Y0*V2+Z0*W2+F0
      Z5=X0*U3+Y0*V3+Z0*W3+G0
      X0=X5
      Y0=Y5
      Z0=Z5
C
C        CONSTRUCT THE COEFFICIENTS OF THE QUARTIC EQUATION:
C              R(5) * X**4 + R(4) * X**3 + R(3) * X**2 +
C              R(2) * X + R(1) = 0
C
      R(5)=1/((A*B)**2)
      R(4)=-2*((1/(A**2))+(1/(B**2)))
      R(3)=4+((B/A)**2)+((A/B)**2)-((X0/B)**2)-((Y0/B)**2)-((Z0/A)**2)
      R(2)=2*(X0**2+Y0**2+Z0**2)-2*(A**2+B**2)
      R(1)=(A*B)**2-(X0*B)**2-(Y0*B)**2-(Z0*A)**2
C
```

```
C          CALL QUART TO SOLVE THE QUARTIC EQUATION FOR ALL
C          REAL ZEROS WHERE:
C                 R = ARRAY OF COEFFICIENTS
C              ROOT = ARRAY OF REAL ZEROS
C                NI = NUMBER OF REAL ZEROS
C
      CALL QUART(R,ROOT,NI)
C
C     DETERMINE THE COORDINATES ON THE ELLIPSOID THAT
C     MAKES THE DISTANCE FROM THE TARGET TO THE SURFACE A MINIMUM.
C
      PS=10.**30
      XJ = 0.0
      YJ = 0.0
      ZJ = 0.0
      DO 16 J=1,NI
      V=ROOT(J)
      X2=(XO*A**2)/(A**2-V)
      Y2=(YO*A**2)/(A**2-V)
      Z2=(ZO*B**2)/(B**2-V)
      XA=(-XO*V)/(A**2-V)
      YA=(-YO*V)/(A**2-V)
      ZA=(-ZO*V)/(B**2-V)
      U=XA**2+YA**2+ZA**2
      U=U**.5
      IF(U.GT.PS) GO TO 16
      PS=U
      XJ=X2
      YJ=Y2
      ZJ=Z2
   16 CONTINUE
      X2=XJ
      Y2=YJ
      Z2=ZJ
C
C          THE ALTITUDE EQUALS THE MINIMUM DISTANCE FROM THE
C          TARGET TO THE SURFACE.
      CON=180./GK
C
C     DETERMINE THE LONGITUDE.
C
      ALONG=ATAN2(YO,XO)
C
C     DETERMINE THE GEODETIC LATITUDE.
C
      D=X2**2+Y2**2
      D=D**.5
      AA=D*(1.-CE)
      ALAT=ATAN2(Z2,AA)
      ALAT=SIGN(ALAT,ZO)
      X1=XO*ZP
      Y1=YO*ZP
      Z1=ZO*ZP
      EX=1./(1.-CE*(SIN(ALAT)**2))
      EX=ABS(EX)
      EX=EX**.5
      EX=EX*A1
```

```
C
C     COMPUTE ALTITUDE BY AN EXACT EQUATION WHICH MINIMIZES ROUND OFF
C     ERRORS.
C
      IF(ABS(SIN(ALAT)).GT..1)GO TO 14
      IF(ABS(COS(ALONG)).GT..1)GO TO 15
      ALT=(Y1/(COS(ALAT)*SIN(ALONG)))-EX
      GO TO 710
   14 ALT=(Z1/SIN(ALAT))-(EX*(1-CE))
      GO TO 710
   15 ALT=(X1/(COS(ALAT)*COS(ALONG)))-EX
  710 CONTINUE
      ALAT=ALAT*CON
C
C     CHANGE SIGN OF LONGITUDE TO CONFORM WITH INPUT WHICH
C     WAS IN DEGREES WEST...CALCULATIONS GIVE LONGITUDE IN
C     DEGREES EAST, CHANGE SIGN TO GET LONGITUDE IN DEGREES
C     WEST, CONVERT FROM RADIANS TO DEGREES.
C
      ALONG=-ALONG*CON
C
      RETURN
      END
```

Subroutine QUART

```
      SUBROUTINE QUART(R,ROOT,NI)
C
C     THIS ROUTINE SOLVES FOR THE REAL ZEROS ONLY VIA
C     FERRARI'S METHOD. (NEW FIRST COURSE IN THE THEORY OF
C     EQUATIONS, DICKSON, PP. 51-52)
C
C
C
C     R IS THE ARRAY WHICH CONTAINS THE COEFFICIENTS OF THE
C     QUARTIC ARRANGED IN ASCENDING ORDER.
C             R(5)  * X**4 + R(4)  * X**3 +
C             R(3)  * X**2 + R(2)  * X + R(1) = 0
C
C     THE ARRAY ROOT WILL CONTAIN THE REAL ZEROS OF THE QUARTIC
C
C     NI IS THE SCALAR VARIABLE THAT STATES THE NUMBER OF REAL ZEROS
C
C
C
      DIMENSION R(1),ROOT(1)
      NI=0
C
C        NORMALIZE COEFFICIENTS SO:
C           X**4 + B*X**3 + C*X**2 + D*X + E = 0
C
      DO 19 J=1,5
   19 R(J)=R(J)/R(5)
      B=R(4)
      C=R(3)
      D=R(2)
      E=R(1)
C
C        CALCULATE COEFFICIENTS B1, C1, D1, OF THE RESOLVENT CUBIC
C           Y**3 + B1*Y**2 + C1*Y + D1 = 0
      B1=-C
      C1=B*D-4*E
      D1=-(B**2)*E+4*C*E-D**2
C
C        IN SOLVING CUBIC EQUATION, CALCULATE COEFFICIENTS
C        OF THE CORRESPONDING REDUCED CUBIC WHICH HAS NO TERM
C        OF THE SECOND DEGREE BY SETTING Y = Z - B1/3!
C           Z**3 + P*Z + Q = 0
C
      P=C1-((B1**2)/3)
      Q=D1-((B1*C1)/3)+((2*(B1**3))/27)
C
C        THE DISCRIMINANT OF THE GENERAL CUBIC EQUATION IS
C        EQUAL TO THE DISCRIMINANT DEL OF THE CORRESPONDING
C        REDUCED EQUATION.
C
      DELT=18*B1*C1*D1-4*(B1**3)*D1+(B1*C1)**2
      DELV=-4*(C1**3)-27*(D1**2)
      DEL=DELT+DELV
C
C        IF DEL IS NEGATIVE, ONE ROOT IS REAL, AND TWO ARE
C        CONJUGATE IMAGINARIES.
C
```

```
      IF(DEL.GE.0.)GO TO 12
      RS=(P/3)**3+(Q/2)**2
      RS=ABS(RS)
      A=((-Q/2)+RS**.5)
      W=((-Q/2)-(RS**.5))
      S=ABS(A)
      T=ABS(W)
      W1=W
      A1=A
      VV=1./3.
      A=S**VV
      W=T**VV
      A=A*SIGN(1.,A1)
      W=W*SIGN(1.,W1)
C
C        Y1 IS THE SINGLE REAL ROOT OF THE REDUCED CUBIC,
C        Y IS THE SINGLE REAL ROOT OF THE GENERAL CUBIC.
C
      Y1=A+W
      Y=Y1-(B1/3)
      GO TO 10
   12 CONTINUE
C
C        IF DEL IS POSITIVE, THERE ARE THREE DISTINCT REAL
C        ROOTS. IF DEL IS ZERO, THERE ARE AT LEAST TWO EQUAL
C        REAL ROOTS.
C               TRIGONOMETRIC SOLUTION IS USED.
C
      TN=(-4*P/3)
      TN=ABS(TN)
      TN=TN**.5
       COS3A=-.5*Q*((-3/P)**1.5)
      IF(ABS(COS3A).GT.1.) COS3A=SIGN(1.,COS3A)
      ARC=ACOS(COS3A)/3
      COSA=COS(ARC)
      Y1=TN*COSA
      Y=Y1-(B1/3)
   10 CONTINUE
C
C        BACK TO SOLVING THE QUARTIC, WHERE Y IS SUCH THAT
C           A2*X**2 + B2*X + C2     IS THE SQUARE OF A LINEAR FUNCTION,
C        M*X + N, AND ALSO EQUAL TO   (X**2 + B*X/2 + Y/2)**2
C
      A2=.25*(B**2)-C+Y
      B2=.5*B*Y-D
      C2=.25*(Y**2)-E
C
      IF(A2.NE.0.)GO TO 111
C
C        IN CASE A2=0,  (X**2 + B*X/2 + Y/2)**2 = B2*X + C2
C        SINCE THE POLYNOMIAL IS A PERFECT SQUARE, B2 = 0
C        AND THE QUADRATICS TO BE SOLVED ARE:
C           X**2 + B*X/2 + Y/2 - C2**.5 = 0    AND
C           X**2 + B*X/2 + Y/2 + C2**.5
C
      DEL1 = (B/2)**2 - 4*(Y/2 - C2**.5)
      DEL2 = (B/2)**2 - 4*(Y/2 + C2**.5)
```

```
          DX1=B/2
          DX2=B/2
          GO TO 223
      111 CONTINUE
C
C            IN CASE  A2  IS NOT 0, THEN   M = A2**.5,   N = B2/(2*M),
C            THEN THE QUADRATICS TO BE SOLVED ARE:
C               X**2 + (B/2 - M)*X + Y/2 - N = 0    AND
C               X**2 + (B/2 + M)*X + Y/2 + N = 0
C
          A2=ABS(A2)
          AM=A2**.5
          AN=B2/(2*AM)
          DX1=.5*B-AM
          DX2=.5*B+AM
          DEL1 = (B/2 - AM)**2 - 4*(Y/2 - AN)
          DEL2 = (B/2 + AM)**2 - 4*(Y/2 + AN)
      223 CONTINUE
C
C            ROOTS OF TWO QUADRATICS ARE THE FOUR ROOTS OF THE QUARTIC,
C                 ONLY THE REAL ROOTS ARE RETURNED.
C
          IF(DEL1.LT.0.)GO TO 22
          NI=NI+1
          ROOT(NI)=(-DX1/2)+((DEL1**.5)/2)
          NI=NI+1
          ROOT(NI)=(-DX1/2)-((DEL1**.5)/2)
       22 IF(DEL2.LT.0.)GO TO 25
          NI=NI+1
          ROOT(NI)=(-DX2/2)+((DEL2**.5)/2)
          NI=NI+1
          ROOT(NI)=(-DX2/2)-((DEL2**.5)/2)
       25 CONTINUE
          RETURN
          END
```

Input and Output Data Samples

```
INPUT-----
          POINT COORDINATES WRT RADAR SITE
                ELEVATION DEG.                          90.0000
                AZI       DEG.                          -0.0000
                RANGE    FEET                             .1000

          RADAR SITE COORDINATES
                GEODETIC LATITUDE                       55.0000
                LONGITUDE                               22.0000
                ALTITUDE      FEET                      -0.0000

OUTPUT----
          POINT COORDINATES
                GEODETIC LATITUDE                       55.0000      .550000000006E+02
                LONGITUDE                               22.0000      .219999999988E+02
                ALTITUDE     FEET                         .0999      .998557806015E-01



INPUT-----
          POINT COORDINATES WRT RADAR SITE
                ELEVATION DEG.                          90.0000
                AZI       DEG.                          -0.0000
                RANGE    FEET                        50000.0000

          RADAR SITE COORDINATES
                GEODETIC LATITUDE                       44.0000
                LONGITUDE                              121.0000
                ALTITUDE      FEET                     500.0000

OUTPUT----
          POINT COORDINATES
                GEODETIC LATITUDE                       44.0000      .440000000264E+02
                LONGITUDE                              121.0000      .120999999999E+03
                ALTITUDE     FEET                    50499.9900      .504999899952E+05



INPUT-----
          POINT COORDINATES WRT RADAR SITE
                ELEVATION DEG.                          90.0000
                AZI       DEG.                          -0.0000
                RANGE    FEET                     999999999.0000

          RADAR SITE COORDINATES
                GEODETIC LATITUDE                       33.0000
                LONGITUDE                               88.0000
                ALTITUDE      FEET                     700.0000

OUTPUT----
          POINT COORDINATES
                GEODETIC LATITUDE                       33.0000      .329999999979E+02
                LONGITUDE                               88.0000      .879999999988E+02
                ALTITUDE     FEET                  1000000699.0492   .100000069905E+10
```

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